■ **Use mathematical induction to prove the following.**

[1] $1 + 4 + 9 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}, \; n \in \mathbb{Z}$

[2] Where $F_1$, $F_2$, etc. are the Fibonacci numbers, prove

$F_1 + F_2 + \cdots + F_n = F_{n+2} - 1$

[3] $(2n)! < (n!)^2 \, 4^{n-1}$ for $\forall \, n \in \mathbb{Z}, \; n \geq 5$

[4] $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{(n-1)n} = \frac{n-1}{n}$ for $n = 2, 3, 4, \ldots$

[5] $6 \mid (n^3 + 5n)$ for $\forall \, n \in \mathbb{Z}^+$

[6] $\frac{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-1)}{2 \cdot 4 \cdot 6 \cdot \ldots \cdot (2n)} \geq \frac{1}{2n}, \; n \in \mathbb{Z}^+$

[7] $\frac{(4n-2)!}{8^n}$ = an integer, for $n = 5, 6, 7, \ldots$

[8] Let $n \in \mathbb{Z}^+$ and $A_1, A_2, \ldots, A_n$ subsets of $\mathbb{U}$.

Prove $\overline{A_1 \cap A_2 \cap A_3 \cap \cdots \cap A_n} = \overline{A_1} \cup \overline{A_2} \cup \cdots \cup \overline{A_n}$

[9] $2 \cdot 6 \cdot 10 \cdot 14 \cdot \ldots \cdot (4n - 2) = \frac{(2n)!}{n!}$

[10] (Bernoulli's Inequality)

Let $a > -1$

Thm $(1 + a)^n \geq 1 + na$, for $n = 0, 1, 2, \ldots, \; a \in$ constant

[11] $1 \cdot 3 \cdot 5 \cdot 7 \cdot \ldots \cdot (2n - 1) \geq 2 \cdot 4 \cdot 6 \cdot \ldots \cdot (2n - 2), \; n \geq 2, \; n \in \mathbb{Z}$

[12] $F_n \geq \left(\frac{5}{4}\right)^n, \; n \geq 3, \; F_n = n$th Fibonacci number

[13] $9 \mid (4^n + 5^n)$, $n \in$ odd

[14] Definition. If $a$ is a real number, then $a^1 = a$ and, for every positive integer $k$, $a^{k+1} = a^k \cdot a$

Prove: $a^m \cdot a^n = a^{m+n}$

[15] (DeMoivre Thm)

If $z = |z| (\cos \phi + i \sin \phi)$, then $z^n = |z|^n (\cos n\phi + i \sin n\phi)$ for each natural number $n$.

[16] $a \equiv b \pmod{m} \Longrightarrow a^k \equiv b^k \pmod{m}$ for each positive integer $k$.